

# PH : PHYSICS

*Duration:* Three Hours

*Maximum Marks:* 100

**Read the following instructions carefully.**

1. Do not open the seal of the Question Booklet until you are asked to do so by the invigilator.
2. Take out the Optical Response Sheet (ORS) from this Question Booklet **without breaking the seal** and read the instructions printed on the ORS carefully.
3. On the right half of the ORS, using ONLY a **black ink ball point pen**, (i) darken the bubble corresponding to your test paper code and the appropriate bubble under each digit of your registration number and (ii) write your registration number, your name and name of the examination centre and put your signature at the specified location.
4. This Question Booklet contains **20** pages including blank pages for rough work. After you are permitted to open the seal, please check all pages and report discrepancies, if any, to the invigilator.
5. There are a total of 65 questions carrying 100 marks. All these questions are of objective type. Each question has only **one** correct answer. Questions must be answered on the left hand side of the ORS by darkening the appropriate bubble (marked A, B, C, D) using ONLY a **black ink ball point pen** against the question number. **For each question darken the bubble of the correct answer.** More than one answer bubbled against a question will be treated as an incorrect response.
6. Since bubbles darkened by the black ink ball point pen **cannot** be erased, candidates should darken the bubbles in the ORS **very carefully**.
7. Questions Q.1 – Q.25 carry 1 mark each. Questions Q.26 – Q.55 carry 2 marks each. The 2 marks questions include two pairs of common data questions and two pairs of linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
8. Questions Q.56 – Q.65 belong to General Aptitude (GA) section and carry a total of 15 marks. Questions Q.56 – Q.60 carry 1 mark each, and questions Q.61 – Q.65 carry 2 marks each.
9. Unattempted questions will result in zero mark and wrong answers will result in **NEGATIVE** marks. For all 1 mark questions,  $\frac{1}{3}$  mark will be deducted for each wrong answer. For all 2 marks questions,  $\frac{2}{3}$  mark will be deducted for each wrong answer. However, in the case of the linked answer question pair, there will be negative marks only for wrong answer to the first question and no negative marks for wrong answer to the second question.
10. Calculator is allowed whereas charts, graph sheets or tables are **NOT** allowed in the examination hall.
11. Rough work can be done on the question paper itself. Blank pages are provided at the end of the question paper for rough work.
12. Before the start of the examination, write your name and registration number in the space provided below using a black ink ball point pen.

Name	Mandeep Singh							
Registration Number	PH	3	0	9	3	1	9	5

## Some Useful Constants

Speed of light in free space	$c = 3 \times 10^8 \text{ m/s}$
Boltzmann constant	$k_B = 1.380 \times 10^{-23} \text{ J/K}$
Planck's constant	$h = 6.626 \times 10^{-34} \text{ J.s}$
Electron charge	$e = 1.602 \times 10^{-19} \text{ C}$
Permittivity of free space	$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N.m}^2$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$

NOTE : In numerical problems, the option closest to the correct answer will be given credit.

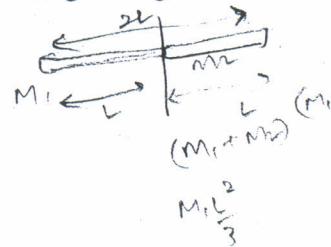
**Q. 1 – Q. 25 carry one mark each.**

Q.1 Identify the CORRECT statement for the following vectors  $\vec{a} = 3\hat{i} + 2\hat{j}$  and  $\vec{b} = \hat{i} + 2\hat{j}$

- (A) The vectors  $\vec{a}$  and  $\vec{b}$  are linearly independent
- (B) The vectors  $\vec{a}$  and  $\vec{b}$  are linearly dependent
- (C) The vectors  $\vec{a}$  and  $\vec{b}$  are orthogonal
- (D) The vectors  $\vec{a}$  and  $\vec{b}$  are normalized

Q.2 Two uniform thin rods of equal length,  $L$ , and masses  $M_1$  and  $M_2$  are joined together along the length. The moment of inertia of the combined rod of length  $2L$  about an axis passing through the mid-point and perpendicular to the length of the rod is,

- (A)  $(M_1 + M_2) \frac{L^2}{12}$
- (B)  $(M_1 + M_2) \frac{L^2}{6}$
- (C)  $(M_1 + M_2) \frac{L^2}{3}$
- (D)  $(M_1 + M_2) \frac{L^2}{2}$



Q.3 The space-time dependence of the electric field of a linearly polarized light in free space is given by  $\hat{x}E_0 \cos(\omega t - kz)$  where  $E_0$ ,  $\omega$  and  $k$  are the amplitude, the angular frequency and the wavevector, respectively. The time averaged energy density associated with the electric field is

- (A)  $\frac{1}{4}\epsilon_0 E_0^2$
- (B)  $\frac{1}{2}\epsilon_0 E_0^2$
- (C)  $\epsilon_0 E_0^2$
- (D)  $2\epsilon_0 E_0^2$

Q.4 If the peak output voltage of a full wave rectifier is 10 V, its d.c. voltage is

- (A) 10.0 V
- (B) 7.07 V
- (C) 6.36 V
- (D) 3.18 V

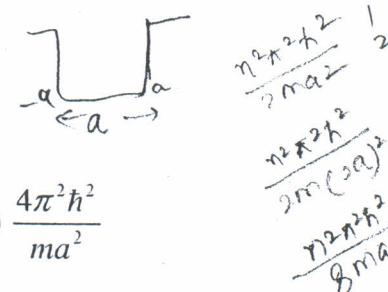
$$\frac{2}{\pi} \cdot \frac{10}{\pi} = \frac{20}{\pi^2}$$

Q5 A particle of mass  $m$  is confined in a two dimensional square well potential of dimension  $a$ . This potential  $V(x,y)$  is given by

$$V(x,y) = 0 \text{ for } -a < x < a \text{ and } -a < y < a \\ = \infty \text{ elsewhere}$$

The energy of the first excited state for this particle is given by,

- (A)  $\frac{\pi^2 \hbar^2}{ma^2}$
- (B)  $\frac{2\pi^2 \hbar^2}{ma^2}$
- (C)  $\frac{5\pi^2 \hbar^2}{2ma^2}$
- (D)  $\frac{4\pi^2 \hbar^2}{ma^2}$



Q.6 The isothermal compressibility,  $\kappa$  of an ideal gas at temperature  $T_0$  and volume  $V_0$ , is given by

(A)  $-\frac{1}{V_0} \frac{\partial V}{\partial P} \Big|_{T_0}$

(B)  $\frac{1}{V_0} \frac{\partial V}{\partial P} \Big|_{T_0}$

(C)  $-V_0 \frac{\partial P}{\partial V} \Big|_{T_0}$

(D)  $V_0 \frac{\partial P}{\partial V} \Big|_{T_0}$

~~Q.7~~ The ground state of sodium atom ( $^{11}\text{Na}$ ) is a  $^2\text{S}_{1/2}$  state. The difference in energy levels arising in the presence of a weak external magnetic field  $B$ , given in terms of Bohr magneton,  $\mu_B$ , is

(A)  $\mu_B B$

(B)  $2\mu_B B$

(C)  $4\mu_B B$

(D)  $6\mu_B B$

Q.8 For an ideal Fermi gas in three dimensions, the electron velocity  $v_F$  at the Fermi surface is related to electron concentration  $n$  as,

(A)  $v_F \propto n^{2/3}$

(B)  $v_F \propto n$

(C)  $v_F \propto n^{1/2}$

(D)  $v_F \propto n^{1/3}$

~~Q.9~~ Which one of the following sets corresponds to fundamental particles?

(A) proton, electron and neutron

(B) proton, electron and photon ✗

(C) electron, photon and neutrino ✗

(D) quark, electron and meson ✗

~~Q.10~~ In case of a Geiger-Muller (GM) counter, which one of the following statements is CORRECT?

(A) Multiplication factor of the detector is of the order of  $10^{10}$

(B) Type of the particles detected can be identified

(C) Energy of the particles detected can be distinguished ✗

(D) Operating voltage of the detector is few tens of Volts ✗

Q.11 A plane electromagnetic wave traveling in free space is incident normally on a glass plate of refractive index  $3/2$ . If there is no absorption by the glass, its reflectivity is

(A) 4%

(B) 16%

(C) 20%

(D) 50%

Q.12 A Ge semiconductor is doped with acceptor impurity concentration of  $10^{15}$  atoms/cm<sup>3</sup>. For the given hole mobility of  $1800 \text{ cm}^2/\text{V-s}$ , the resistivity of this material is

(A)  $0.288 \Omega \text{ cm}$

(B)  $0.694 \Omega \text{ cm}$

(C)  $3.472 \Omega \text{ cm}$

(D)  $6.944 \Omega \text{ cm}$

~~Q.13~~ A classical gas of molecules, each of mass  $m$ , is in thermal equilibrium at the absolute temperature,  $T$ . The velocity components of the molecules along the Cartesian axes are  $v_x$ ,  $v_y$  and  $v_z$ . The mean value of  $(v_x + v_y)^2$  is

(A)  $\frac{k_B T}{m}$

(B)  $\frac{3 k_B T}{2 m}$

(C)  $\frac{1}{2} \frac{k_B T}{m}$

(D)  $\frac{2 k_B T}{m}$

Q.14 In a central force field, the trajectory of a particle of mass  $m$  and angular momentum  $L$  in plane polar coordinates is given by,

$$\frac{1}{r} = \frac{m}{L^2} (1 + \epsilon \cos \theta)$$

where,  $\epsilon$  is the eccentricity of the particle's motion. Which one of the following choices for  $\epsilon$  gives rise to a parabolic trajectory?

(A)  $\epsilon = 0$

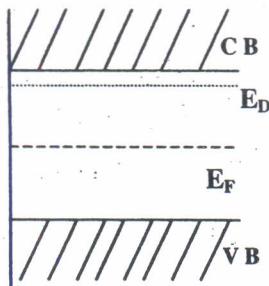
(B)  $\epsilon = 1$

(C)  $0 < \epsilon < 1$

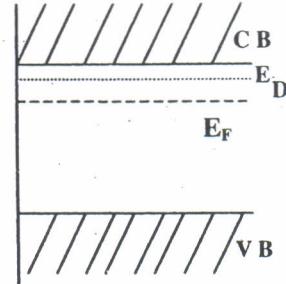
(D)  $\epsilon > 1$

Q.15 Identify the CORRECT energy band diagram for Silicon doped with Arsenic. Here CB, VB, E<sub>D</sub> and E<sub>F</sub> are conduction band, valence band, impurity level and Fermi level, respectively.

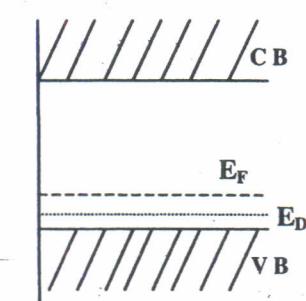
(A)



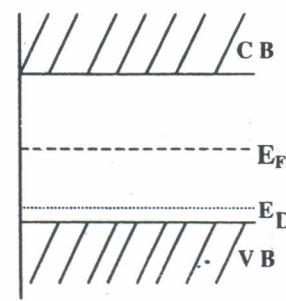
(B)



(C)



(D)



Q.16 The first Stokes line of a rotational Raman spectrum is observed at  $12.96 \text{ cm}^{-1}$ . Considering the rigid rotor approximation, the rotational constant is given by

- (A)  $6.48 \text{ cm}^{-1}$       (B)  $3.24 \text{ cm}^{-1}$       (C)  $2.16 \text{ cm}^{-1}$       (D)  $1.62 \text{ cm}^{-1}$        $B = \frac{12.96}{6}$

Q.17 The total energy,  $E$  of an ideal non-relativistic Fermi gas in three dimensions is given by

$$E \propto \frac{N^{\frac{5}{3}}}{V^{\frac{2}{3}}} \quad \text{where } N \text{ is the number of particles and } V \text{ is the volume of the gas.}$$

Identify the CORRECT equation of state ( $P$  being the pressure),

- (A)  $PV = \frac{1}{3}E$       (B)  $PV = \frac{2}{3}E$       (C)  $PV = E$       (D)  $PV = \frac{5}{3}E$

Q.18 Consider the wavefunction  $\Psi = \psi(\vec{r}_1, \vec{r}_2)\chi_s$  for a fermionic system consisting of two spin-half particles. The spatial part of the wavefunction is given by,

$$\psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}} [\phi_1(\vec{r}_1)\phi_2(\vec{r}_2) + \phi_2(\vec{r}_1)\phi_1(\vec{r}_2)]$$

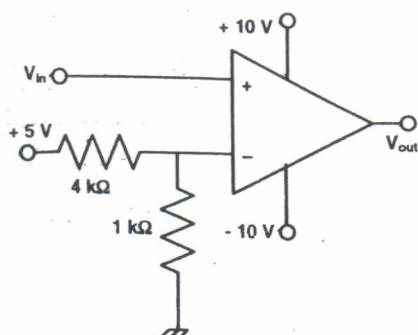
where  $\phi_1$  and  $\phi_2$  are single particle states. The spin part  $\chi_s$  of the wavefunction with spin states  $\alpha(+\frac{1}{2})$  and  $\beta(-\frac{1}{2})$  should be

- (A)  $\frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha)$       (B)  $\frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha)$       (C)  $\alpha\alpha$       (D)  $\beta\beta$

Q.19 The electric and the magnetic fields  $\vec{E}(z, t)$  and  $\vec{B}(z, t)$ , respectively corresponding to the scalar potential  $\phi(z, t) = 0$  and vector potential  $\vec{A}(z, t) = \hat{i}tz$  are

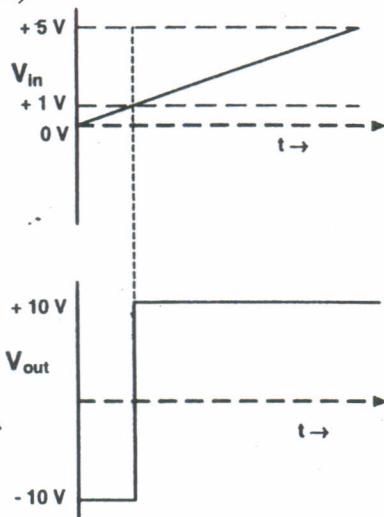
- (A)  $\vec{E} = \hat{i}z$  and  $\vec{B} = -\hat{j}t$       (B)  $\vec{E} = \hat{i}z$  and  $\vec{B} = \hat{j}t$   
 (C)  $\vec{E} = -\hat{i}z$  and  $\vec{B} = -\hat{j}t$       (D)  $\vec{E} = -\hat{i}z$  and  $\vec{B} = \hat{j}t$

**Q20** Consider the following OP-AMP circuit.

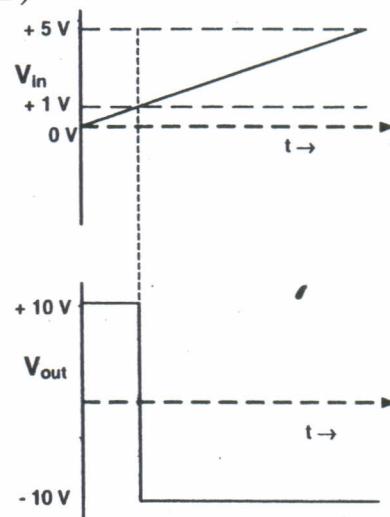


Which one of the following correctly represents the output  $V_{out}$  corresponding to the input  $V_{in}$ ?

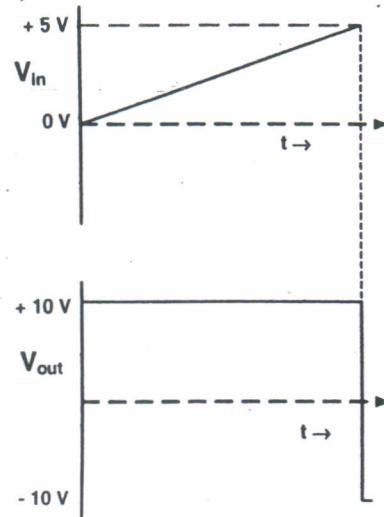
(A)



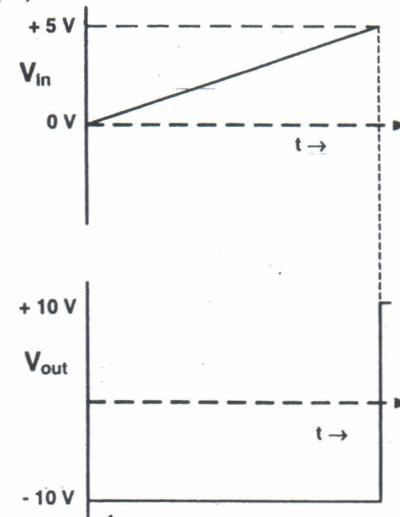
(B)



(C)



(D)



**Q21**

Deuteron has only one bound state with spin parity  $1^+$ , isospin 0 and electric quadrupole moment 0.286 efm $^2$ . These data suggest that the nuclear forces are having

**Q20**

- (A) only spin and isospin dependence
- (B) no spin dependence and no tensor components
- (C) spin dependence but no tensor components
- (D) spin dependence along with tensor components

$$V(x) = x(x^2 + 4 - 4x)$$

$$V(x) = x^3 + 4x - 4x^2$$

$$V''(x) = 3x^2 + 4 - 8x$$

$$V''(x) = 12 - 8x$$

PHYSICS - PH

- Q.22 A particle of unit mass moves along the  $x$ -axis under the influence of a potential,  $V(x) = x(x-2)^2$ . The particle is found to be in stable equilibrium at the point  $x=2$ . The time period of oscillation of the particle is

(A)  $\frac{\pi}{2}$

(B)  $\pi$

(C)  $\frac{3\pi}{2}$

(D)  $2\pi$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2k}{m}}} = \pi\sqrt{\frac{m}{k}}$

- Q.23 Which one of the following CANNOT be explained by considering a harmonic approximation for the lattice vibrations in solids?

(A) Debye's  $T^3$  law

(B) Dulong Petit's law

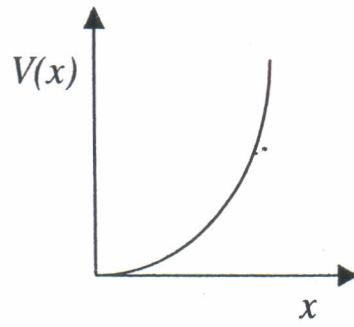
(C) Optical branches in lattices

(D) Thermal expansion

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi/2} = \omega = 2\pi$

$T =$

- Q.24 A particle is constrained to move in a truncated harmonic potential well ( $x > 0$ ) as shown in the figure. Which one of the following statements is CORRECT?



$T = \frac{9\pi}{2\omega^2}$

$= \frac{9\pi}{\omega}$

$\omega^2 = k$

$= \infty \frac{\partial K}{\partial x}$

(A) The parity of the first excited state is even

(B) The parity of the ground state is even

(C) The ground state energy is  $\frac{1}{2}\hbar\omega$

(D) The first excited state energy is  $\frac{7}{2}\hbar\omega$

$2x_1 + \frac{3}{2}\hbar$

- Q.25 The number of independent components of the symmetric tensor  $A_{ij}$  with indices  $i, j = 1, 2, 3$  is

(A) 1

(B) 3

(C) 6

(D) 9

$\frac{3(4)}{2}^2$

**Q. 26 to Q. 55 carry two marks each.**

- Q.26 Consider a system in the unperturbed state described by the Hamiltonian,  $H_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ . The system

$|1\rangle \langle 0|$

is subjected to a perturbation of the form  $H' = \begin{pmatrix} \delta & \delta \\ \delta & \delta \end{pmatrix}$ , where  $\delta \ll 1$ . The energy eigenvalues of

$|1\rangle \langle 1|$

the perturbed system using the first order perturbation approximation are

$E_n^{(1)} = \langle 1 | H' | 1 \rangle$

(A) 1 and  $(1+2\delta)$

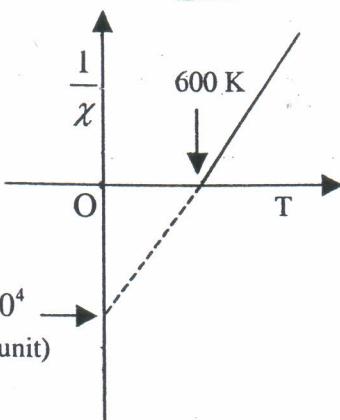
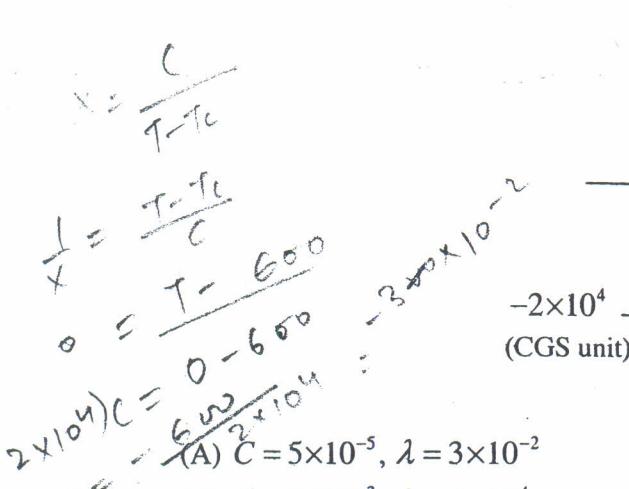
(B)  $(1+\delta)$  and  $(1-\delta)$

(C)  $(1+2\delta)$  and  $(1-2\delta)$

(D)  $(1+\delta)$  and  $(1-2\delta)$

Q.27

Inverse susceptibility ( $1/\chi$ ) as a function of temperature,  $T$  for a material undergoing paramagnetic to ferromagnetic transition is given in the figure, where O is the origin. The values of the Curie constant,  $C$ , and the Weiss molecular field constant,  $\lambda$ , in CGS units, are



$$(T-T_c)C = \frac{1}{\chi}$$

$$(600-T_c)C = 0$$

$$T_c = 600^\circ K$$

Q.28 A plane polarized electromagnetic wave in free space at time  $t=0$  is given by  $\vec{E}(x, z) = 10\hat{j}\exp[i(6x + 8z)]$ . The magnetic field  $\vec{B}(x, z, t)$  is given by

(A)  $\vec{B}(x, z, t) = \frac{1}{c}(6\hat{k} - 8\hat{i})\exp[i(6x + 8z - 10ct)]$

(B)  $\vec{B}(x, z, t) = \frac{1}{c}(6\hat{k} + 8\hat{i})\exp[i(6x + 8z - 10ct)]$

(C)  $\vec{B}(x, z, t) = \frac{1}{c}(6\hat{k} - 8\hat{i})\exp[i(6x + 8z - ct)]$

(D)  $\vec{B}(x, z, t) = \frac{1}{c}(6\hat{k} + 8\hat{i})\exp[i(6x + 8z + ct)]$

$$\vec{B} = \frac{\vec{E} \times \vec{k}}{(\epsilon_0 \omega)^2}$$

$$\vec{k} = \vec{R} \times \vec{E}$$

$$\vec{E} = \frac{2 \times 10^4}{600} \hat{i}$$

$$= 3.33 \times 10^3 \hat{i}$$

Q.29 The eigenvalues of the matrix  $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$  are

(A) 0, 1, 1

(B)  $0, -\sqrt{2}, \sqrt{2}$

(C)  $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$

(D)  $\sqrt{2}, \sqrt{2}, 0$

Q.30 Match the typical spectroscopic regions specified in Group I with the corresponding type of transitions in Group II.

### Group I

(P) Infra-red region

(Q) Ultraviolet-visible region

(R) X-ray region

(S)  $\gamma$ -ray region

(A) (P, i); (Q, iii); (R, ii); (S, iv)

(Q) (P, iii); (Q, i); (R, iv); (S, ii)

### Group II

(i) electronic transitions involving valence electrons

(ii) nuclear transitions

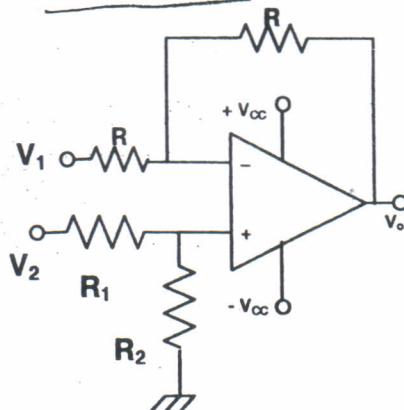
(iii) vibrational transitions of molecules

(iv) transitions involving inner shell electrons

(B) (P, ii); (Q, iv); (R, i); (S, iii)

(D) (P, iv); (Q, i); (R, ii); (S, iii)

Q.31 In the following circuit, for the output voltage to be  $V_o = (-V_1 + V_2/2)$ , the ratio  $R_1/R_2$  is



(A) 1/2

(B) 1

(C) 2

(D) 3

Q.32 The terms  $\{j_1, j_2\}$ , arising from  $2s^1 3d^1$  electronic configuration in j-j coupling scheme are

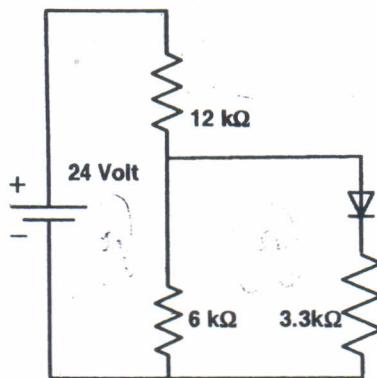
(A)  $\left\{\frac{1}{2}, \frac{3}{2}\right\}_{2,1}$  and  $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$

(C)  $\left\{\frac{1}{2}, \frac{1}{2}\right\}_{1,0}$  and  $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$

(B)  $\left\{\frac{1}{2}, \frac{1}{2}\right\}_{1,0}$  and  $\left\{\frac{1}{2}, \frac{3}{2}\right\}_{2,1}$

(D)  $\left\{\frac{3}{2}, \frac{1}{2}\right\}_{2,1}$  and  $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$

Q.33 In the following circuit, the voltage drop across the ideal diode in forward bias condition is 0.7 V.



The current passing through the diode is

(A) 0.5 mA

(B) 1.0 mA

(C) 1.5 mA

(D) 2.0 mA

Q.34 Choose the CORRECT statement from the following.

(A) Neutron interacts through electromagnetic interaction

(B) Electron does not interact through weak interaction

(C) Neutrino interacts through weak and electromagnetic interaction

(D) Quark interacts through strong interaction but not through weak interaction

2013

Q.35 A rod of proper length  $l_0$  oriented parallel to the  $x$ -axis moves with speed  $2c/3$  along the  $x$ -axis in the  $S$ -frame, where  $c$  is the speed of the light in free space. The observer is also moving along the  $x$ -axis with speed  $c/2$  with respect to the  $S$ -frame. The length of the rod as measured by the observer is

(A)  $0.35 l_0$ (B)  $0.48 l_0$ (C)  $0.87 l_0$ (D)  $0.97 l_0$

- Q.36 A simple cubic crystal with lattice parameter  $a_c$  undergoes transition into a tetragonal structure with lattice parameters  $a_t = b_t = \sqrt{2} a_c$  and  $c_t = 2a_c$ , below a certain temperature. The ratio of the interplanar spacings of (101) planes for the cubic and the tetragonal structures is

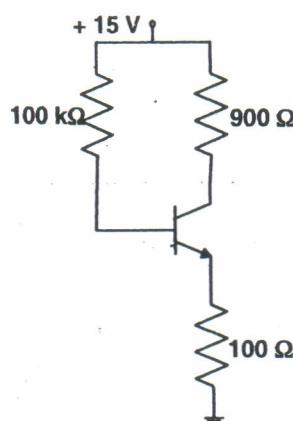
(A)  $\sqrt{\frac{1}{6}}$

(B)  $\frac{1}{6}$

(C)  $\sqrt{\frac{3}{8}}$

(D)  $\frac{3}{8}$

- Q.37 Consider the following circuit in which the current gain  $\beta_{dc}$  of the transistor is 100.

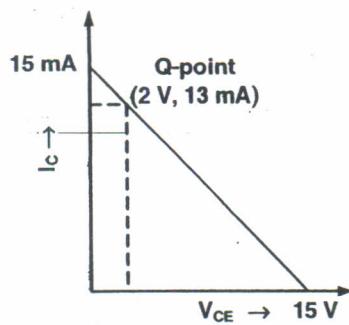


$$\frac{1}{1 + \frac{900}{100}}$$

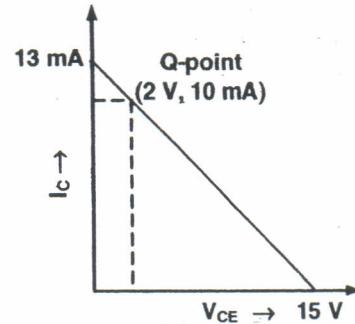
$$\frac{a_c}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2a_c} = \frac{\sqrt{3}}{2\sqrt{2}}$$

Which one of the following correctly represents the load line (collector current  $I_C$  with respect to collector-emitter voltage  $V_{CE}$ ) and Q-point of this circuit?

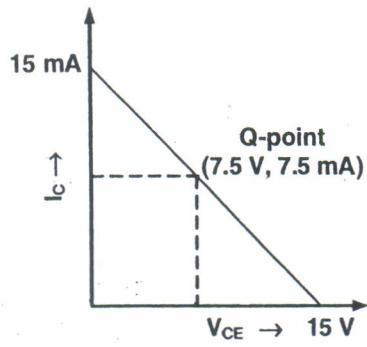
(A)



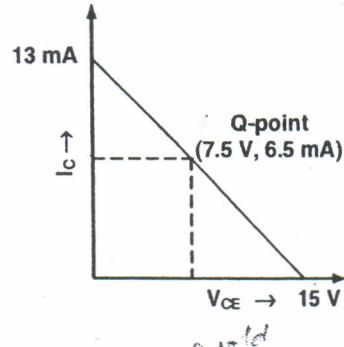
(B)



(C)



(D)



- Q.38 Consider a system whose three energy levels are given by 0,  $\varepsilon$  and  $2\varepsilon$ . The energy level  $\varepsilon$  is two-fold degenerate and the other two are non-degenerate. The partition function of the system

with  $\beta = \frac{1}{k_B T}$  is given by

(A)  $1 + 2e^{-\beta\varepsilon}$

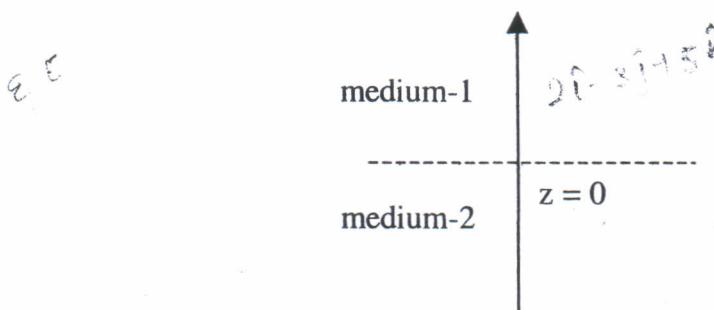
(C)  $(1 + e^{-\beta\varepsilon})^2$

(B)  $2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$

(D)  $1 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$

- Q.39 Two infinitely extended homogeneous isotropic dielectric media (medium-1 and medium-2 with dielectric constants  $\frac{\epsilon_1}{\epsilon_0} = 2$  and  $\frac{\epsilon_2}{\epsilon_0} = 5$ , respectively) meet at the  $z = 0$  plane as shown in the figure. A uniform electric field exists everywhere. For  $z \geq 0$ , the electric field is given by  $\vec{E}_1 = 2\hat{i} - 3\hat{j} + 5\hat{k}$ . The interface separating the two media is charge free.

The electric displacement vector in the medium-2 is given by



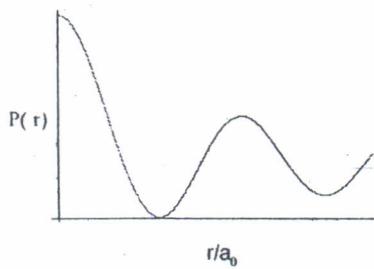
$$\begin{aligned} D_1^2 - D_2^2 &= \frac{Q}{\epsilon_0} \\ \epsilon_1 D_1^2 - \epsilon_2 D_2^2 &= Q \\ \frac{2}{3} D_1^2 - \frac{5}{3} D_2^2 &= Q \\ D''_1 - D''_2 &= Q \end{aligned}$$

- (A)  $\vec{D}_2 = \epsilon_0[10\hat{i} + 15\hat{j} + 10\hat{k}]$   
 (B)  $\vec{D}_2 = \epsilon_0[10\hat{i} - 15\hat{j} + 10\hat{k}]$   
(C)  $\vec{D}_2 = \epsilon_0[4\hat{i} - 6\hat{j} + 10\hat{k}]$   
(D)  $\vec{D}_2 = \epsilon_0[4\hat{i} + 6\hat{j} + 10\hat{k}]$

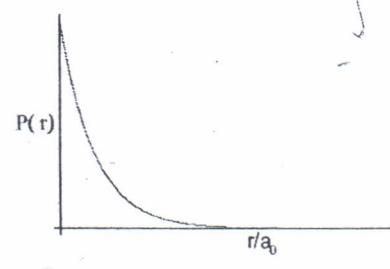
- Q.40 The ground state wavefunction for the hydrogen atom is given by  $\psi_{100} = \frac{1}{\sqrt{4\pi}} \left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$ , where  $a_0$  is the Bohr radius.

The plot of the radial probability density,  $P(r)$  for the hydrogen atom in the ground state is

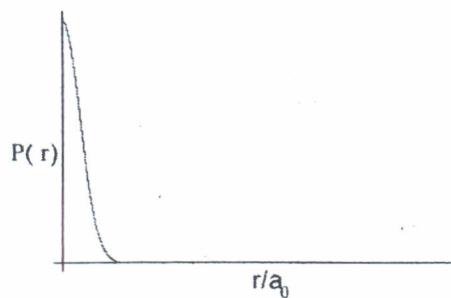
(A)



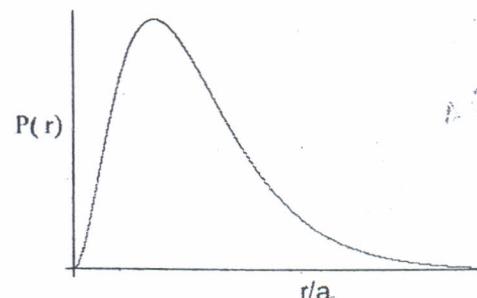
(B)



(C)



(D)



- Q.41 Total binding energies of  $O^{15}$ ,  $O^{16}$  and  $O^{17}$  are  $111.96 \text{ MeV}$ ,  $127.62 \text{ MeV}$  and  $131.76 \text{ MeV}$ , respectively. The energy gap between  $1p_{1/2}$  and  $1d_{5/2}$  neutron shells for the nuclei whose mass number is close to 16, is

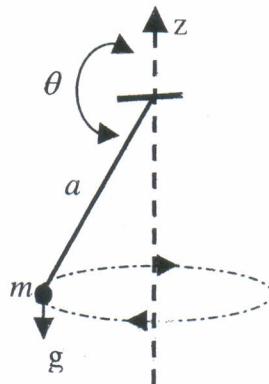
- (A)  $4.1 \text{ MeV}$       (B)  $11.5 \text{ MeV}$       (C)  $15.7 \text{ MeV}$       (D)  $19.8 \text{ MeV}$

- Q.42 A particle of mass  $m$  is attached to a fixed point O by a weightless inextensible string of length  $a$ . It is rotating under the gravity as shown in the figure.

The Lagrangian of the particle is

$$L(\theta, \phi) = \frac{1}{2}ma^2(\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2) - mga \cos \theta$$

where  $\theta$  and  $\phi$  are the polar angles.

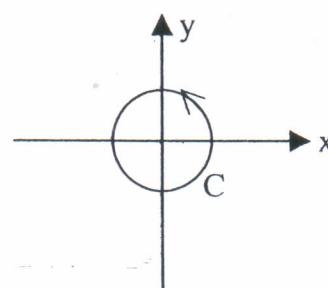


The Hamiltonian of the particle is

(A)  $H = \frac{1}{2ma^2} \left( p_\theta^2 + \frac{p_\phi^2}{\sin^2 \theta} \right) - mga \cos \theta$       (B)  $H = \frac{1}{2ma^2} \left( p_\theta^2 + \frac{p_\phi^2}{\sin^2 \theta} \right) + mga \cos \theta$

(C)  $H = \frac{1}{2ma^2} (p_\theta^2 + p_\phi^2) - mga \cos \theta$       (D)  $H = \frac{1}{2ma^2} (p_\theta^2 + p_\phi^2) + mga \cos \theta$

- Q.43 Given  $\vec{F} = \vec{r} \times \vec{B}$ , where  $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})$  is a constant vector and  $\vec{r}$  is the position vector. The value of  $\oint_C \vec{F} \cdot d\vec{r}$ , where  $C$  is a circle of unit radius centered at origin is,



(A) 0

(B)  $2\pi B_0$

(C)  $-2\pi B_0$

(D) 1

- Q.44 The value of the integral  $\oint_C e^{yz} dz$ , using the contour  $C$  of circle with unit radius  $|z|=1$  is

(A) 0

(B)  $1-2\pi i$

(C)  $1+2\pi i$

(D)  $2\pi i$

- Q.45 A paramagnetic system consisting of  $N$  spin-half particles, is placed in an external magnetic field. It is found that  $N/2$  spins are aligned parallel and the remaining  $N/2$  spins are aligned antiparallel to the magnetic field. The statistical entropy of the system is,

(A)  $2Nk_B \ln 2$

(B)  $\frac{N}{2} k_B \ln 2$

(C)  $\frac{3N}{2} k_B \ln 2$

(D)  $Nk_B \ln 2$

w<sub>c</sub> 2990

- Q.46** The equilibrium vibration frequency for an oscillator is observed at  $2990 \text{ cm}^{-1}$ . The ratio of the frequencies corresponding to the first and the fundamental spectral lines is 1.96. Considering the oscillator to be anharmonic, the anharmonicity constant is

(A) 0.005      (B) 0.02      (C) 0.05      (D) 0.1

- Q.47** At a certain temperature  $T$ , the average speed of nitrogen molecules in air is found to be 400 m/s. The most probable and the root mean square speeds of the molecules are, respectively,

(A) 355 m/s, 434 m/s      (B) 820 m/s, 917 m/s  
 (C) 152 m/s, 301 m/s      (D) 422 m/s, 600 m/s

### Common Data Questions

#### Common Data for Questions 48 and 49:

The wavefunction of a particle moving in free space is given by,  $\psi = e^{ikx} + 2e^{-ikx}$

- Q.48** The energy of the particle is

(A)  $\frac{5\hbar^2 k^2}{2m}$       (B)  $\frac{3\hbar^2 k^2}{4m}$   
 (C)  $\frac{\hbar^2 k^2}{2m}$       (D)  $\frac{\hbar^2 k^2}{m}$

$$\frac{V_{avg}}{V_{RMS}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\sqrt{\frac{5}{3}} = \frac{2}{\sqrt{3}}$$

- Q.49** The probability current density for the real part of the wavefunction is

(A) 1      (B)  $\frac{\hbar k}{m}$       (C)  $\frac{\hbar k}{2m}$       (D) 0

$$\frac{V_{avg}}{V_{RMS}} = \frac{2}{\sqrt{3}}$$

$$\frac{\sqrt{4/3}}{2\sqrt{5/3}} = \frac{\sqrt{4/3}}{\sqrt{5/3}} = \frac{2}{\sqrt{5}}$$

### Common Data for Questions 50 and 51:

The dispersion relation for a one dimensional monatomic crystal with lattice spacing  $a$ , which interacts via nearest neighbour harmonic potential is given by

$$\omega = A \left| \sin \frac{Ka}{2} \right|$$

where  $A$  is a constant of appropriate unit.

- Q.50** The group velocity at the boundary of the first Brillouin zone is

(A) 0      (B) 1      (C)  $\sqrt{\frac{Aa^2}{2}}$       (D)  $\frac{1}{2}\sqrt{\frac{Aa^2}{2}}$

- Q.51** The force constant between the nearest neighbour of the lattice is ( $M$  is the mass of the atom)

(A)  $\frac{MA^2}{4}$       (B)  $\frac{MA^2}{2}$       (C)  $MA^2$       (D)  $2MA^2$

W/A

### Linked Answer Questions

#### Statement for Linked Answer Questions 52 and 53:

In a hydrogen atom, consider that the electronic charge is uniformly distributed in a spherical volume of radius  $a (= 0.5 \times 10^{-10} \text{ m})$  around the proton. The atom is placed in a uniform electric field  $E = 30 \times 10^5 \text{ V/m}$ . Assume that the spherical distribution of the negative charge remains undistorted under the electric field.

- Q.52 In the equilibrium condition, the separation between the positive and the negative charge centers is

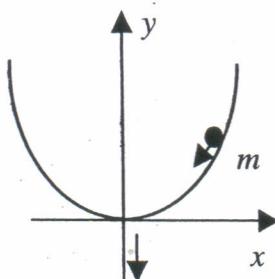
(A)  $8.66 \times 10^{-16} \text{ m}$       (B)  $2.60 \times 10^{-15} \text{ m}$       (C)  $2.60 \times 10^{-16} \text{ m}$       (D)  $8.66 \times 10^{-15} \text{ m}$

- Q.53 The polarizability of the hydrogen atom in unit of ( $\text{C}^2\text{m/N}$ ) is

(A)  $2.0 \times 10^{-40}$       (B)  $1.4 \times 10^{-41}$       (C)  $1.4 \times 10^{-40}$       (D)  $2.0 \times 10^{-39}$

#### Statement for Linked Answer Questions 54 and 55:

A particle of mass  $m$  slides under the gravity without friction along the parabolic path  $y = ax^2$  as shown in the figure. Here  $a$  is a constant.



- Q.54 The Lagrangian for this particle is given by,

(A)  $L = \frac{1}{2}m\dot{x}^2 - mgax^2$

(B)  $L = \frac{1}{2}m(1+4a^2x^2)\dot{x}^2 - mgax^2$

(C)  $L = \frac{1}{2}m\dot{x}^2 + mgax^2$

(D)  $L = \frac{1}{2}m(1+4a^2x^2)\dot{x}^2 + mgax^2$

- Q.55 The Lagrange's equation of motion of the particle is

(A)  $\ddot{x} = 2gax$

(B)  $m(1+4a^2x^2)\ddot{x} = -2mgax - 4ma^2x\dot{x}^2$

(C)  $m(1+4a^2x^2)\ddot{x} = 2mgax + 4ma^2x\dot{x}^2$

(D)  $\ddot{x} = -2gax$

$$\frac{2990}{2} - \frac{2990 \times e}{4} = \frac{2990}{2} - \frac{2990 \times e}{4}$$

$$\frac{3 \times 2990}{2} - \frac{9 \times 2990 \times e}{8} = 0.96$$

$$\frac{1995 - 747.5 \times e}{4485 - 6727.5 \times e}$$

$$\frac{1248.5 \times e}{1243.8 \times e} = 9295.6$$

$$e = 0.9665$$

## General Aptitude (GA) Questions

**Q. 56 – Q. 60 carry one mark each.**

Q.56 Choose the grammatically **INCORRECT** sentence:

- (A) They gave us the money back less the service charges of Three Hundred rupees.
- (B) This country's expenditure is not less than that of Bangladesh.
- (C) The committee initially asked for a funding of Fifty Lakh rupees, but later settled for a lesser sum.
- (D) This country's expenditure on educational reforms is very less.

Q.57 Which one of the following options is the closest in meaning to the word given below?

**Mitigate**

- (A) Diminish      (B) Divulge      (C) Dedicate      (D) Denote

Q.58 Choose the most appropriate alternative from the options given below to complete the following sentence:

Despite several \_\_\_\_\_ the mission succeeded in its attempt to resolve the conflict.

- (A) attempts      (B) setbacks      (C) meetings      (D) delegations

Q.59 The cost function for a product in a firm is given by  $5q^2$ , where  $q$  is the amount of production. The firm can sell the product at a market price of ₹ 50 per unit. The number of units to be produced by the firm such that the profit is maximized is

- (A) 5      (B) 10      (C) 15      (D) 25

Q.60 Choose the most appropriate alternative from the options given below to complete the following sentence:

Suresh's dog is the one \_\_\_\_\_ was hurt in the stampede.

- (A) that      (B) which      (C) who      (D) whom

**Q. 61 - Q. 65 carry two marks each.**

Q.61 Which of the following assertions are **CORRECT**?

- P: Adding 7 to each entry in a list adds 7 to the mean of the list
- Q: Adding 7 to each entry in a list adds 7 to the standard deviation of the list
- R: Doubling each entry in a list doubles the mean of the list
- S: Doubling each entry in a list leaves the standard deviation of the list unchanged

- (A) P, Q      (B) Q, R      (C) P, R      (D) R, S

- Q.62 An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable.

The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is

- (A) 0.288      (B) 0.334      (C) 0.667      (D) 0.720

- Q.63 A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation  $y = 2x - 0.1x^2$  where  $y$  is the height of the arch in meters. The maximum possible height of the arch is

- (A) 8 meters      (B) 10 meters      (C) 12 meters      (D) 14 meters

- Q.64 Wanted Temporary, Part-time persons for the post of Field Interviewer to conduct personal interviews to collect and collate economic data. Requirements: High School-pass, must be available for Day, Evening and Saturday work. Transportation paid, expenses reimbursed.

Which one of the following is the best inference from the above advertisement?

- (A) Gender-discriminatory  
 (B) Xenophobic  
 (C) Not designed to make the post attractive  
 (D) Not gender-discriminatory

- Q.65 Given the sequence of terms, AD CG FK JP, the next term is

- (A) OV      (B) OW      (C) PV      (D) PW

### END OF THE QUESTION PAPER